SEPTEMBER 21, 2017

LAST CLASS

<u>Pumping Lemma</u> $\Rightarrow$ Let A be a regular language $\exists$ pumping length $p>0$ s.t.

$\forall s \in \Sigma^*$ satisfying:

(a) $s \in A$ and

(b) $|s| \geq p$

it holds that $\exists x, y, z$ s.t. $s = xyz$:

① $\forall i \geq 0 \quad xy^iz \in A$

② $|y| > 0$

③ $|xy| \leq p$

<u>Example</u> Let $B = \{ww \mid w \in \{0,1\}^*\}$. Claim B is not regular.

Pf/ By contradiction. Assume B is regular

Let p be the pumping length for B

Let $s = \underbrace{0^p1}_{w}\underbrace{0^p1}_{w}$ Then ⓐ $s \in B$ and ⓑ $|s| = 2p + 2 \geq p$

$\therefore \exists x, y, z$ s.t. $s = xyz$ and:

By ③ : $|xy| \leq p \Rightarrow y$ consists only of zeroes

By ② : $|y| > 0 \Rightarrow y$ contains at least one zero

By ① : $xy^2z = 0^{p'}10^p1$ for $p' > p$

$xy^2z \notin B$ $\perp$ contradiction. ∎

<u>Example</u> Let $B = \{0^i1^j \mid i > j\}$ B is not regular

Pf/ By contradiction. Assume B is regular with pumping length p.

Let $s = 0^{p+1}1^p$. Then ⓐ $s \in B$ and ⓑ $|s| = 2p+1 \geq p$

0^p1^{p-1} $\therefore \exists x, y, z$ s.t. $s = xyz$ and:

$|xy| \leq p$ By ③ : $|xy| \leq p \Rightarrow y$ contains only zeroes

By ② : $|y| > 0 \Rightarrow y$ contains at least one zero

By ① : $xy^0z = xz = 0^{p'}1^p$ for $p' \leq p$

$xy^0z \notin B$ $\perp$ ∎

context-free languages $\Leftarrow$ the next level

DFA

$\boxed{2.1 \Rightarrow \text{in text}}$

context-free grammar (CFG)

Terminals

ex: G : $\boxed{A} \to 0A1$

$A \to B$ ⟵ substitution rules

$B \to \varepsilon$

start variable.

capital letters: variables

⇒ How to generate a string with a CFG:

① start with start variable

② while you have variables left:

pick a variable, replace it with a substitution rule.

e.g. $A \to 0A1 \to 0B1 \to 0\varepsilon 1 \to \boxed{01}$

e.g. $A \to B \to \boxed{\varepsilon}$

- The sequence of substitutions to obtain a string is called a derivation

⇒ context-free grammar (CFG)

A CFG is a 4-tuple (V, Σ, R, S) s.t.

① V is a finite set of variables

② Σ is a finite set of terminals s.t. $V \cap \Sigma = \emptyset$

③ R is a finite set of substitution rules, where a rule has a variable of left and variables / terminals on right

④ $S \in V$ is start variables

More terminology

$u \Rightarrow v$ means u "yields" v, i.e. applying 1 subst rule to u gives v.

$u \overset{*}{\Rightarrow} v$ means u derives v, i.e. either:

① $u = v$, or

② $\exists$ sequence of strings $(u_1, \ldots, u_k)$ for $k \geq 0$ s.t. $u \Rightarrow u_1 \Rightarrow u_2 \Rightarrow \ldots \Rightarrow u_k \Rightarrow v$

So language of G, $L(G) = \{x \in \Sigma^* \mid S \overset{*}{\Rightarrow} x\}$

Example 2.3 $G = (\{S\}, \{(,)\}, R, S\}$ where rules R are:

$S \to (S)$

$S \to SS$ $\equiv$ $S \to (S) \mid SS \mid \varepsilon$

$S \to \varepsilon$

$L(G) = \{$ set of all strings of balanced parentheses $\}$

e.g. $()$, ε, $(()())$, $(()(())$ X

Example 2.4 $G = (V, \Sigma, R, A)$ where
$V = \{A, B, C\}$
$\Sigma = \{a, +, \times, (,)\}$
Rules R: $A \longrightarrow A + B \mid B$
$B \longrightarrow B \times C \mid C$
$C \longrightarrow (A) \mid a$

$\underline{a + a \times a}$:
$A \Rightarrow A + B$
$\Rightarrow A + B \times C$
$\Rightarrow A + C \times a$
$\Rightarrow A + a \times a$
$\Rightarrow B + a \times a$
$\Rightarrow a + a \times a.$